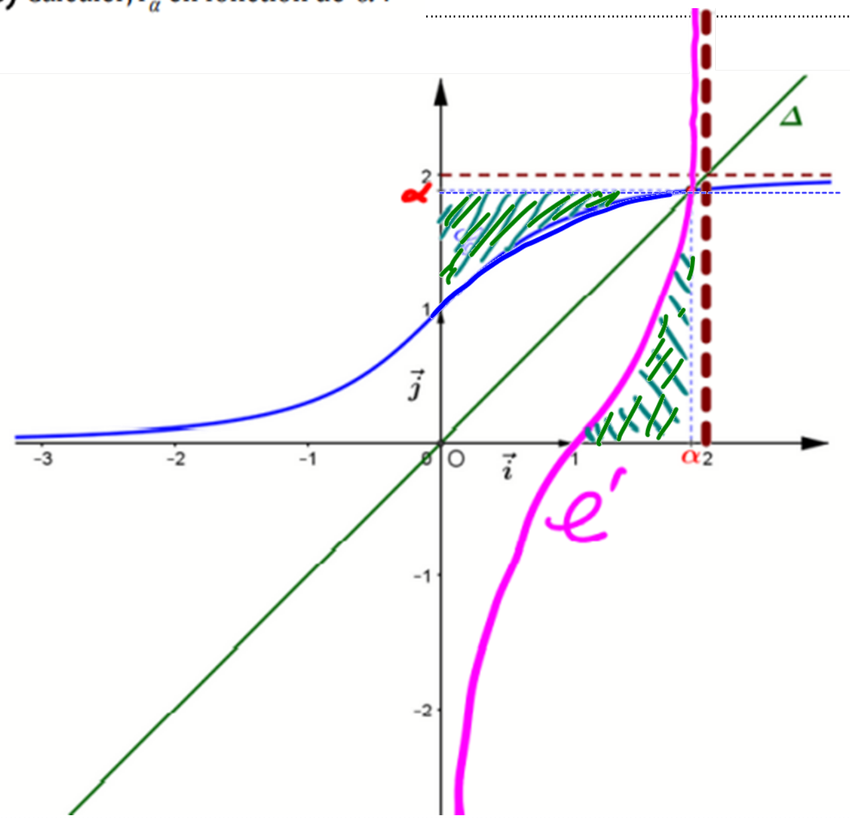
b) Calculer, I_α en fonction de α.



$$I_\alpha = \int_1^\alpha f^{-1}(x)\,dx$$

. par éléments de symétrie on a

$$\int_1^\alpha f^{-1}(x)\,dx = \int_0^\alpha (\alpha - f(x))\,dx$$

$$= \int_0^\alpha \alpha\,dx - \int_0^\alpha f(x)\,dx$$

$$= \alpha(\alpha - 0) - \frac{\alpha^2 - \alpha + 1}{\alpha - 1}$$

$$= \frac{\alpha^2(\alpha-1) - \alpha^2 + \alpha - 1}{\alpha - 1}$$

$$\boxed{I_\alpha = \frac{\alpha^3 - 2\alpha^2 + \alpha - 1}{\alpha - 1}}$$